



CWI

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Are we modeling the wrong stress tensor in LES?

In collaboration with Benjamin Sanderse and Roel Verstappen



Stress in LES

S. D. Agdestein

Turbulence

Lifecycle of turbulence

Distribution of energy

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The Burgers' equation (1D)

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$$\partial_t u = \underbrace{\nu \partial_{xx} u}_{\text{Diffusion}} - \underbrace{\partial_x (u^2/2)}_{\text{Convection}}$$



The Burgers' equation (1D)

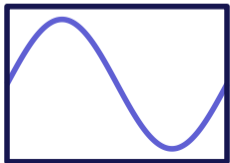
Stress in LES

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Effect on a Fourier mode

$$u = e^{ikx}$$





The Burgers' equation (1D)

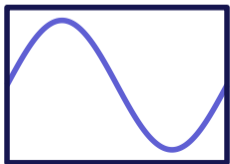
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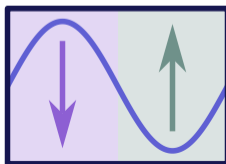
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$$\partial_{xx} u = -k^2 e^{ikx}$$





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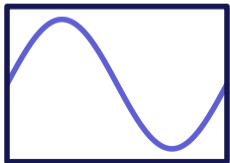
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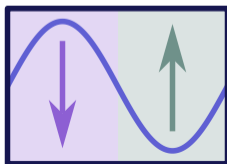
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Effect on a Fourier mode

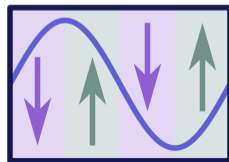
$$u = e^{ikx}$$



$$\partial_{xx} u = -k^2 e^{ikx}$$



$$\partial_x (u^2/2) = ike^{i(2k)x}$$





Spectral distribution of energy

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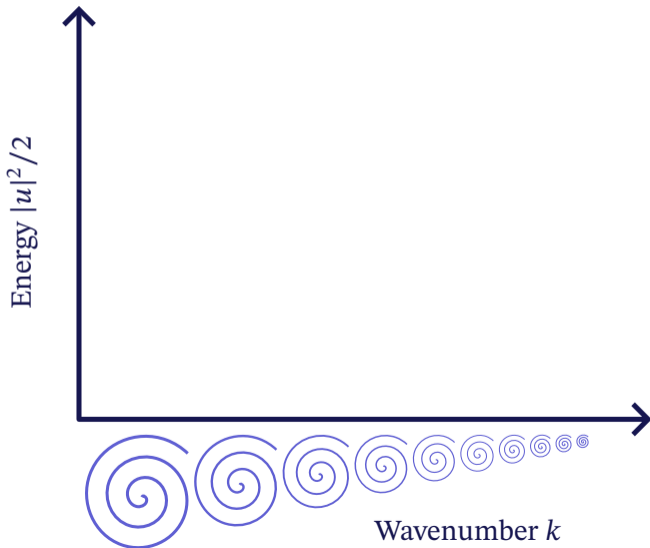
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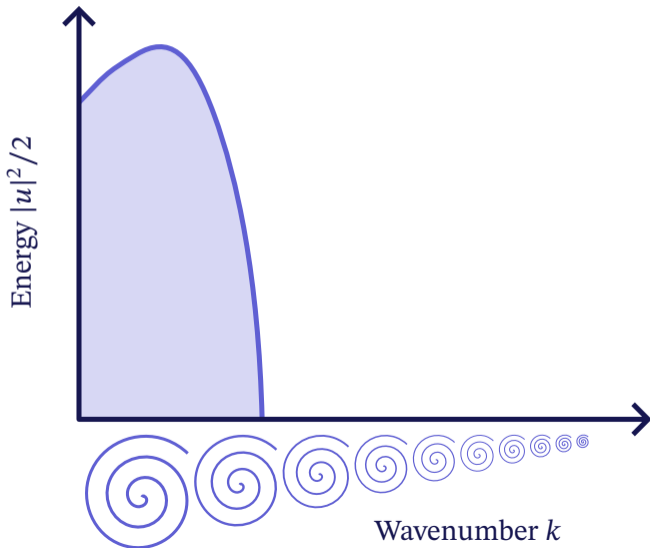
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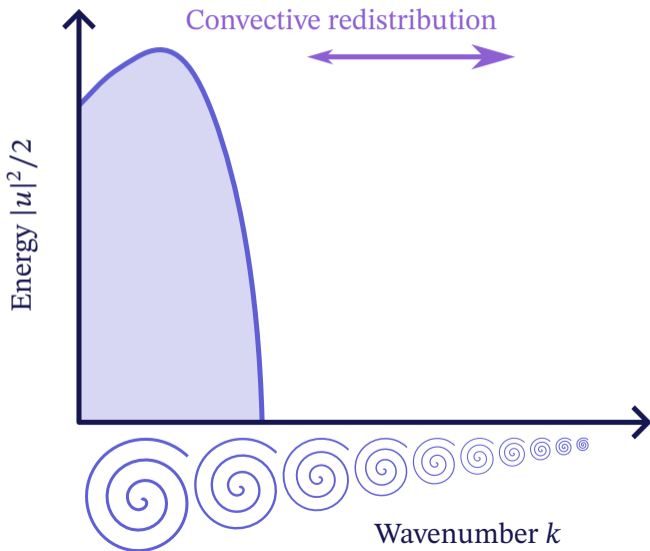
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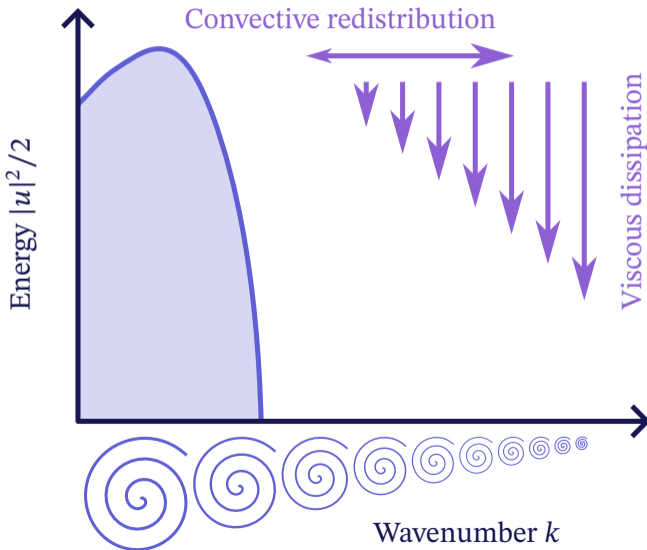
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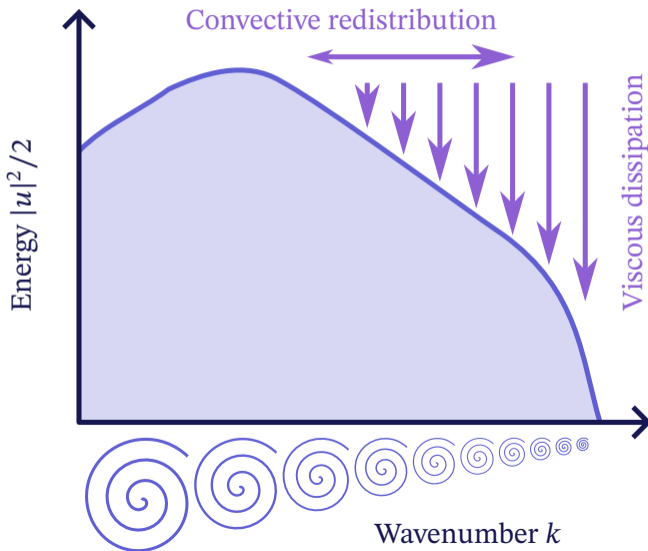
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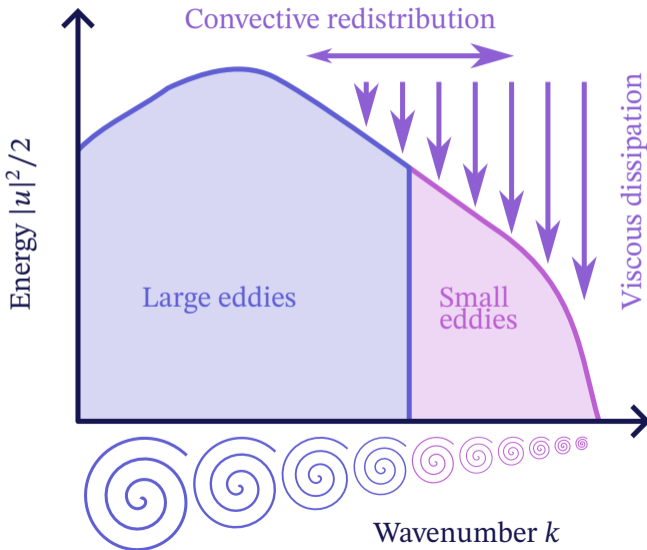
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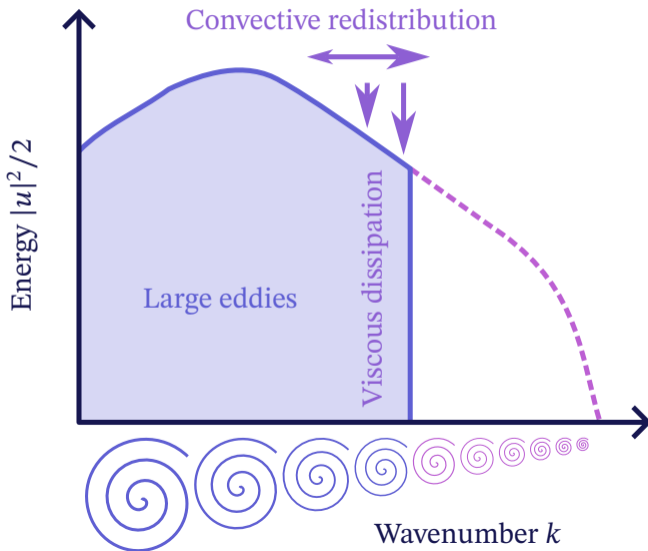
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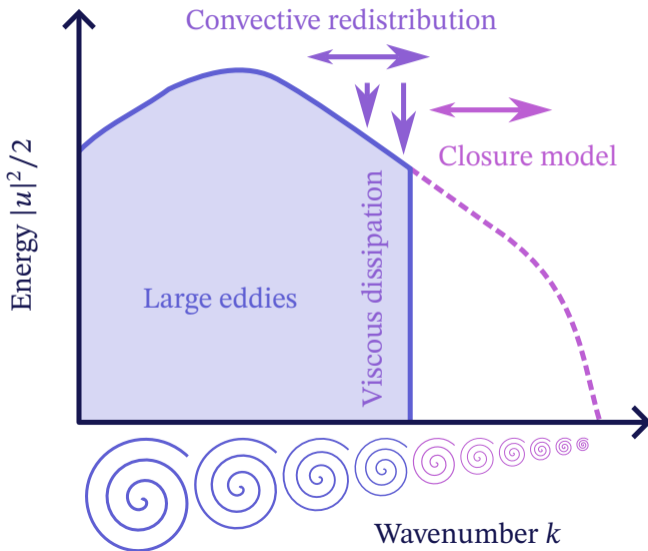
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LES and FVM: the toolbox

Filtering

$$\overline{(\cdot)}^{\Delta}, \overline{(\cdot)}^h$$



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LES and FVM: the toolbox

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$$\overline{(\cdot)}^{\Delta}, \overline{(\cdot)}^h$$

Divergence theorem

$$\partial_x^h = \overline{(\cdot)}^h \circ \partial_x$$



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$$\overline{(\cdot)}^{\Delta}, \overline{(\cdot)}^h$$

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$$\tau^{\Delta}(u) \approx m^{\Delta}(\bar{u}^{\Delta})$$

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$$\sigma(u) \approx \sigma^h(\bar{u}^h)$$



Filtering

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Pressure projection

$$\operatorname{div} \circ \pi = 0, \quad \pi \pi = \pi$$

Divergence theorem

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Flux reconstruction

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Time integration

$$u^* = u^n + \Delta t \dots$$
$$u^{n+1} = \pi u^*$$



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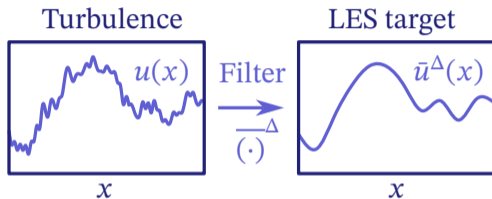


$$\bar{u}^\Delta(x) = \int_{\Omega} G^\Delta(x - y)u(y) dy$$



LES filter

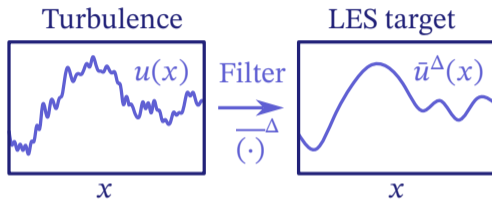
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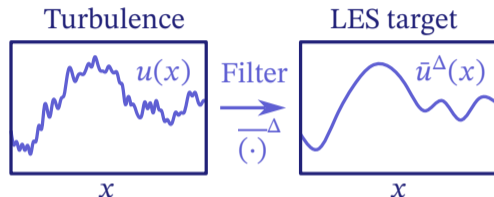
FVM filter

$$\bar{u}^h(x) = \frac{1}{h} \int_{x-h/2}^{x+h/2} u(y) dy$$



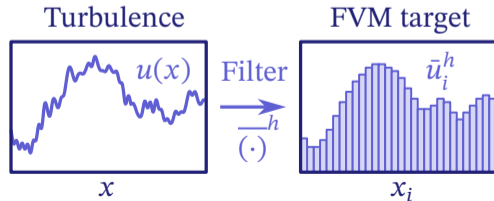
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The mechanism powering the FVM



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The mechanism powering the FVM

$$\partial_x^h u(x) = \frac{u\left(x + \frac{h}{2}\right) - u\left(x - \frac{h}{2}\right)}{h} = \frac{1}{h} \int_{x-h/2}^{x+h/2} \partial_y u(y) dy = \overline{\partial_x u}^h(x)$$



Divergence theorem

The mechanism powering the FVM

$$\partial_x^h u(x) = \frac{u\left(x + \frac{h}{2}\right) - u\left(x - \frac{h}{2}\right)}{h} = \frac{1}{h} \int_{x-h/2}^{x+h/2} \partial_y u(y) dy = \overline{\partial_x u}^h(x)$$

The discrete divergence ∂_x^h is induced by the FVM filter $\overline{(\cdot)}^h$
 \implies **exact** discrete conservation laws for \bar{u}^h



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Models rely on τ^Δ being the unresolved stress



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Closure modeling

Models rely on τ^Δ being the unresolved stress

$$\tau^\Delta(u) = \overline{uu}^\Delta - \bar{u}^\Delta \bar{u}^\Delta, \quad S(u) = (\nabla u + \nabla u^T)/2$$



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Structural models

$$m^\Delta(\bar{u}^\Delta) = \frac{\Delta^2}{12} (\nabla \bar{u}^\Delta)(\nabla \bar{u}^\Delta)^T, \quad (\text{Taylor series truncation})$$

$$m^\Delta(\bar{u}^\Delta) = \tau^{(2\Delta)}(\bar{u}^\Delta), \quad (\text{scale-similarity})$$



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Functional models

$$m^\Delta(\bar{u}^\Delta) = -2\nu^\Delta S(\bar{u}^\Delta), \quad 2\nu_{\text{ideal}}^\Delta = -\frac{S(\bar{u}^\Delta) : \tau^\Delta(u)}{S(\bar{u}^\Delta) : S(\bar{u}^\Delta)}, \quad (\text{eddy-viscosity})$$



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Infinitesimal flux

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$$\sigma(u) = uu$$



Infinitesimal flux

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$$\sigma(u) = uu$$

Discrete flux

$$\sigma^h(u)(x) = \left[\frac{u\left(x + \frac{h}{2}\right) + u\left(x - \frac{h}{2}\right)}{2} \right]^2 \quad (\text{energy-conserving central})$$
$$\sigma^h(u)(x) = \begin{cases} u\left(x - \frac{h}{2}\right)^2, & \text{if } u\left(x - \frac{h}{2}\right) > 0, \\ u\left(x + \frac{h}{2}\right)^2, & \text{if } u\left(x + \frac{h}{2}\right) < 0, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{dissipative upwind})$$

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Projection and time integration

Incompressible Navier-Stokes as a conservation law



Projection and time integration

Incompressible Navier-Stokes as a conservation law

With pressure and constraint

$$\nabla \cdot u = 0, \quad \partial_t u + \nabla \cdot (\sigma(u) + p\delta) = f, \quad \sigma(u) = uu - \nu(\nabla u + \nabla u^T)$$

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Pressure-free formulation

$$\partial_t u + \nabla \cdot \pi\sigma(u) = \pi f$$



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Incompressible Navier-Stokes as a conservation law

With pressure and constraint

$$\nabla \cdot \mathbf{u} = 0, \quad \partial_t \mathbf{u} + \nabla \cdot (\boldsymbol{\sigma}(\mathbf{u}) + p\boldsymbol{\delta}) = \mathbf{f}, \quad \boldsymbol{\sigma}(\mathbf{u}) = \mathbf{u}\mathbf{u} - \nu(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$$

Pressure-free formulation

$$\partial_t \mathbf{u} + \nabla \cdot \boldsymbol{\pi}\boldsymbol{\sigma}(\mathbf{u}) = \boldsymbol{\pi}\mathbf{f}$$

Time integration

$$\begin{aligned} \mathbf{u}^* &= \mathbf{u}^n + \Delta t(\mathbf{f} - \nabla \cdot \boldsymbol{\sigma}(\mathbf{u}^n)) \\ \mathbf{u}^{n+1} &= \boldsymbol{\pi}\mathbf{u}^* \end{aligned}$$



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Discretization-aware LES

Correcting for an inconsistency



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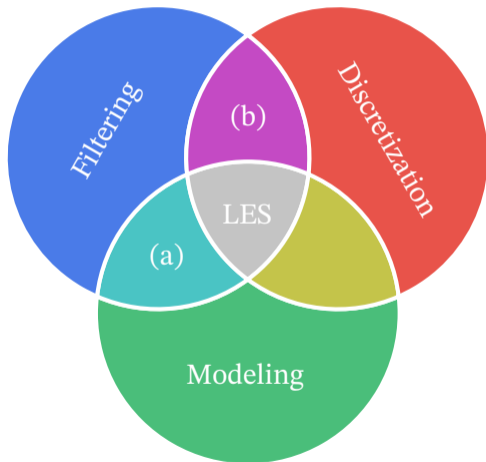
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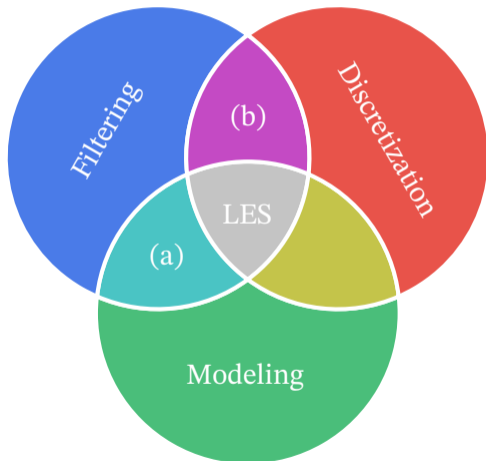
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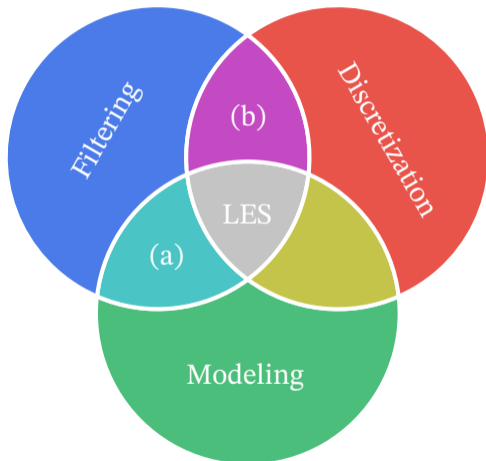


- a** Classical approach: Filter and model, then discretize.



Discretization-aware LES

Correcting for an inconsistency



- a** Classical approach:
Filter and model,
then discretize.
- b** New approach:
Filter and discretize,
then model.



Burgers' equation

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$$\partial_t u + \partial_x \sigma(u) = 0, \quad \sigma(u) = uu/2 - \nu \partial_x u$$



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$$\partial_t u + \partial_x \sigma(u) = 0, \quad \sigma(u) = uu/2 - \nu \partial_x u$$

Apply LES/FVM filters

$$\partial_t \bar{u}^{\Delta, h} + \partial_x^h \overline{\sigma(u)}^\Delta = 0$$

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Apply LES/FVM filters

$$\partial_t \bar{u}^{\Delta, h} + \partial_x^h \overline{\sigma(u)}^{\Delta} = 0$$

Introduce numerical flux

$$\partial_t \bar{u}^{\Delta, h} + \partial_x^h \sigma^h(\bar{u}^{\Delta, h}) = -\partial_x^h \tau^{\Delta, h}(u)$$



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Apply LES/FVM filters

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Introduce numerical flux

$$\partial_t \bar{u}^{\Delta,h} + \partial_x^h \sigma^h(\bar{u}^{\Delta,h}) = -\partial_x^h \tau^{\Delta,h}(u)$$

Unresolved discrete flux

$$\tau^{\Delta,h}(u) = \overline{\sigma(u)}^\Delta - \sigma^h(\bar{u}^{\Delta,h})$$



Incompressible Navier-Stokes

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Projection form

$$\partial_t u = -\nabla \cdot \pi \sigma(u), \quad \sigma(u) = uu - \nu (\nabla u + \nabla u^T)$$



Projection form

$$\partial_t u = -\nabla \cdot \pi \sigma(u), \quad \sigma(u) = uu - \nu (\nabla u + \nabla u^T)$$

Exact LES equations

$$\text{Infinitesimal LES: } \partial_t \bar{u} = -\overline{\nabla \cdot \pi \sigma(u)} = -\nabla \cdot \overline{\pi \sigma(u)}$$

$$\text{Discrete LES: } \partial_t \bar{u}^h = -\overline{\nabla \cdot \pi \sigma(u)}^h = -\nabla_h \cdot \overline{\pi \sigma(u)}^{h,*}$$



Projection form

$$\partial_t u = -\nabla \cdot \pi \sigma(u), \quad \sigma(u) = uu - \nu (\nabla u + \nabla u^T)$$

Exact LES equations

Infinitesimal LES: $\partial_t \bar{u} = -\overline{\nabla \cdot \pi \sigma(u)} = -\nabla \cdot \overline{\pi \sigma(u)}$

Discrete LES: $\partial_t \bar{u}^h = -\overline{\nabla \cdot \pi \sigma(u)}^h = -\nabla_h \cdot \overline{\pi \sigma(u)}^{h,*}$

τ_{11}	τ_{12}	τ_{13}
τ_{12}	τ_{22}	τ_{23}
τ_{13}	τ_{23}	τ_{33}

τ_{11}	τ_{12}	τ_{13}
τ_{21}	τ_{22}	τ_{23}
τ_{31}	τ_{32}	τ_{33}

- Turbulence
- LES components
- Discrete LES
- Inconsistency
- Burgers' equation
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Discretization contribution

How much of the sub-filter flux comes from the discretization?

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Discretization contribution

How much of the sub-filter flux comes from the discretization?

$$\tau^{\Delta,h}(\mathbf{u}) = \tau_{\text{classic}}^{\Delta,h}(\mathbf{u}) + \tau_{\text{flux}}^{\Delta,h}(\mathbf{u}) + \tau_{\text{div}}^{\Delta,h}(\mathbf{u})$$



How much of the sub-filter flux comes from the discretization?

$$\tau^{\Delta,h}(\mathbf{u}) = \tau_{\text{classic}}^{\Delta,h}(\mathbf{u}) + \tau_{\text{flux}}^{\Delta,h}(\mathbf{u}) + \tau_{\text{div}}^{\Delta,h}(\mathbf{u})$$

$$\tau_{\text{classic}}^{\Delta,h}(\mathbf{u}) := \overline{\sigma(\mathbf{u})}^{\Delta,h} - \sigma(\bar{\mathbf{u}}^{\Delta,h})$$

$$\tau_{\text{flux}}^{\Delta,h}(\mathbf{u}) := \sigma(\bar{\mathbf{u}}^{\Delta,h}) - \sigma^h(\bar{\mathbf{u}}^{\Delta,h})$$

$$\tau_{\text{div}}^{\Delta,h}(\mathbf{u}) := \overline{\sigma(\mathbf{u})}^{\Delta} - \overline{\sigma(\mathbf{u})}^{\Delta,h}$$



Discretization contribution

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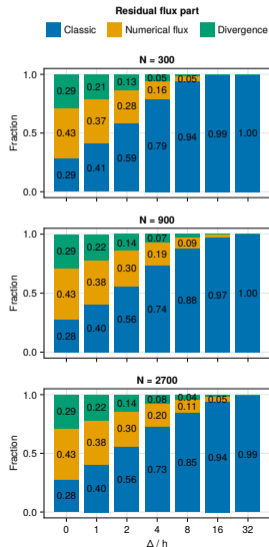
Conclusion

$$\tau^{\Delta,h}(u) = \tau_{\text{classic}}^{\Delta,h}(u) + \tau_{\text{flux}}^{\Delta,h}(u) + \tau_{\text{div}}^{\Delta,h}(u)$$

$$\tau_{\text{classic}}^{\Delta,h}(u) := \overline{\sigma(u)^{\Delta,h}} - \sigma(\bar{u}^{\Delta,h})$$

$$\tau_{\text{flux}}^{\Delta,h}(u) := \sigma(\bar{u}^{\Delta,h}) - \sigma^h(\bar{u}^{\Delta,h})$$

$$\tau_{\text{div}}^{\Delta,h}(u) := \overline{\sigma(u)^{\Delta}} - \overline{\sigma(u)^{\Delta,h}}$$





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$$m(u, \theta) = -\theta s(u), \quad m^h(u, \theta) = -\theta s^h(u),$$

$$s(u) = |\partial_x u| \partial_x u, \quad s^h(u) = |\partial_x^h u| \partial_x^h u,$$



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$$\theta_{\text{classic}} = -\frac{s(\bar{u})\tau(u)}{s(\bar{u})s(\bar{u})},$$

$$\theta_{\text{informed}} = -\frac{s^h(\bar{u})\tau^h(u)}{s^h(\bar{u})s^h(\bar{u})}.$$



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Comparison:

$m^h(\cdot, \theta_{\text{classic}})$ vs $m^h(\cdot, \theta_{\text{informed}})$



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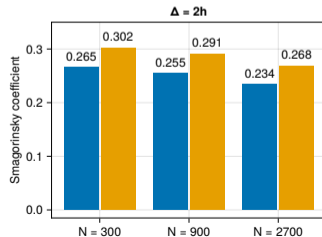
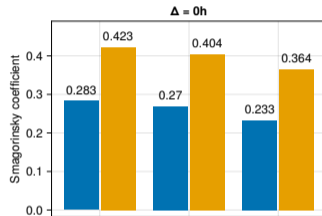
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Classic Discretization-informed (ours)





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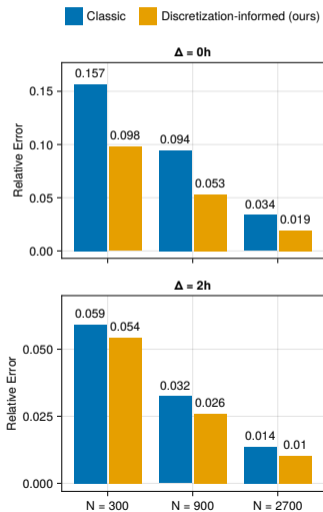
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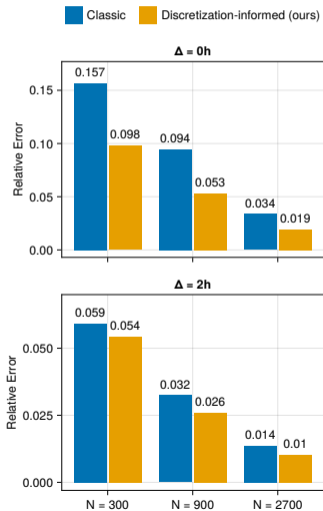
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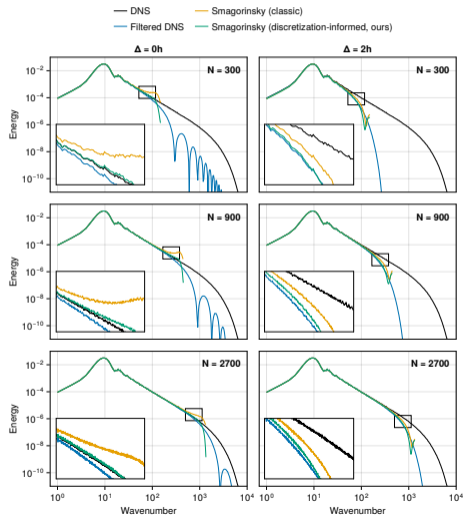
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Stress expressions for incompressible Navier-Stokes



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LES equation

$$\partial_t \bar{u}^h = -\nabla_h \cdot \pi^h [\sigma^h(\bar{u}^h) + \tau^h(u)]$$

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Five stress candidates

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➤ τ_A^h : No SFS

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Five stress candidates

- τ_A^h : No SFS
- τ_B^h : Classical SFS

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LES equation

$$\partial_t \bar{u}^h = -\nabla_h \cdot \pi^h [\sigma^h(\bar{u}^h) + \tau^h(u)]$$

Five stress candidates

- › τ_A^h : No SFS
- › τ_B^h : Classical SFS
- › τ_C^h : Classical SFS with num. flux



LES equation

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- › τ_D^h : Correct SFS



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- τ_D^h : Correct SFS
- τ_E^h : Correct SFS (symmetrized)

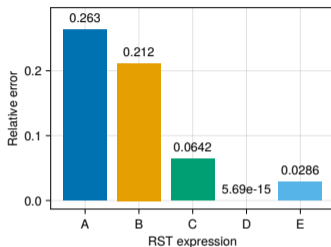


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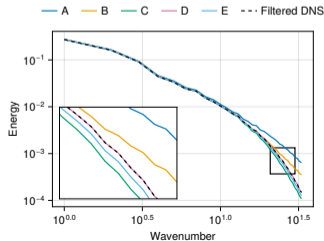
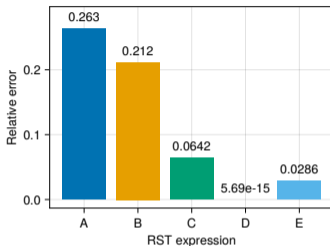


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Thank you!

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Thank you!

The discretization is a fundamental part of LES.



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The discretization is a fundamental part of LES.

*For incompressible Navier-Stokes, the LES-FVM stress tensor is **non-local** and **non-symmetric**.*



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The discretization is a fundamental part of LES.

*For incompressible Navier-Stokes, the LES-FVM stress tensor is **non-local** and **non-symmetric**.*

*Infinitesimal structural closure models have the **wrong structure**.*



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Implicit and explicit LES unified into one coherent framework.



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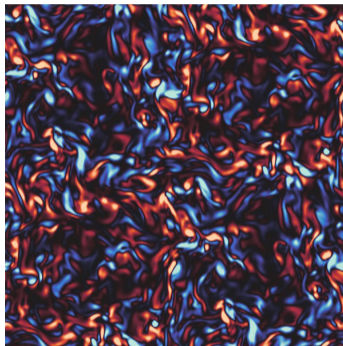
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<https://agdestein.github.io/>